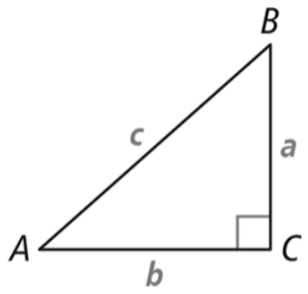
Section 8.1 Pythagorean Theorem and Special Right Triangles

Pythagorean Theorem

If a triangle is a right triangle, then the sum of the squares of the lengths of the legs is equal to the square of the length of the hypotenuse.

PROOF: SEE EXAMPLE 1.

If... $\triangle ABC$ is a right triangle.



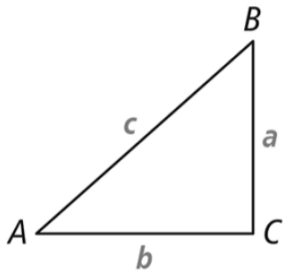
Then... $a^2 + b^2 = c^2$

Converse of the Pythagorean Theorem

If the sum of the squares of the lengths of two sides of a triangle is equal to the square of the length of the third side, then the triangle is a right triangle.

PROOF: SEE EXERCISE 17.

If... $a^2 + b^2 = c^2$



Then... $\triangle ABC$ is a right triangle.

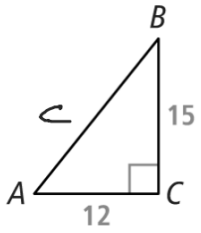
Comparing a^2, b^2, and c^2

$a^2 + b^2 = c^2$ Right Triangle

$a^2 + b^2 > c^2$ Acute Triangle

$a^2 + b^2 < c^2$ Obtuse Triangle

Use the figure shown. Find *AB*.



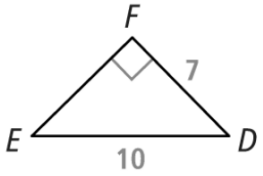
$12^2 + 15^2 = c^2$

$144 + 225 = c^2$

$369 = c^2$

$c = \sqrt{369} \approx 19.2$

Use the figure shown. Find *EF*.



$a^2 + 7^2 = 10^2$

$a^2 + 49 = 100$

$a^2 = 51$

$a = \sqrt{51} \approx 7.141$

Classify the triangle with the given sides as acute, obtuse, or right.

2, 3, 4

$2^2+3^2 \stackrel{?}{=} 4^2$

$4+9$

$13 < 16$

obtuse

3, 4, 5

$3^2+4^2 \stackrel{?}{=} 5^2$

$9+16$

$25 = 25$

Right

4, 5, 6

$4^2+5^2 \quad 6^2$

$16+25$

$41 > 36$

acute

1.2, 2.0, 2.5

$1.2^2+2.0^2 \quad 2.5^2$

$1.44+4 \quad 6.25$

$5.44 < 6.25$

obtuse

0.6, 0.8, 0.9

$(.6)^2+(.8)^2 \quad (.9)^2$

$.36+.64 \quad .81$

$1 > .81$

Acute

1, 1, $\sqrt{2}$

$1^2+1^2 \quad (\sqrt{2})^2$

$1+1 = 2$

$2 = 2$

Right

1, $\sqrt{2}$, 2

$1^2+(\sqrt{2})^2 \quad 2^2$

$1+2 \quad 4$

$3 < 4$

obtuse

$\sqrt{2}$, 2, $\sqrt{5}$

$(\sqrt{2})^2+2^2 \quad (\sqrt{5})^2$

$2+4 \quad 5$

$6 > 5$

Acute

11, 14, 17

11, 12, 17

12, 35, 37

1, 3,

A. To satisfy safety regulations, the distance from the wall to the base of a ladder should be at least one-fourth the length of the ladder. Did Drew set up the ladder correctly?

SOLUTION

$(2.5)^2 + 9^2 = (\text{Ladder})^2$

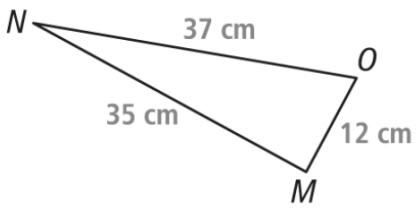
$6.25 + 81 = L^2$

$87.25 = L^2$

$L = 9.34\text{ ft}$

$\hookrightarrow 2.33$

Is $\triangle MNO$ a right triangle? Explain.



$12^2 + 35^2 \stackrel{?}{=} 37^2$

$144 + 1225 = 1369$

$1369 \stackrel{\checkmark}{=} 1369$

Yes $\triangle MNO$ is a Rt $\triangle$

Special Right Triangles

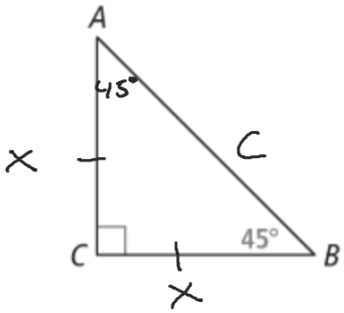
45°- 45° - 90°

Is there a relationship between the lengths of $\overline{AB}$ and $\overline{AC}$ in $\triangle ABC$? Explain.

Legs are congruent

Hypotenuse = Leg $\cdot \sqrt{2}$

SOLUTION



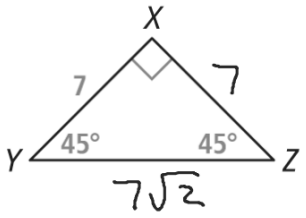
$x^2 + x^2 = c^2$

$\sqrt{2x^2} = \sqrt{c^2}$

$c = x\sqrt{2}$

3. Find the side lengths of the 45°-45°-90° triangle.

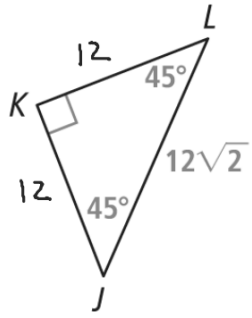
a. What are *XZ* and *YZ*?



3. Find the side lengths of the 45°-45°-90° triangle.

b. What are *JK* and *LK*?

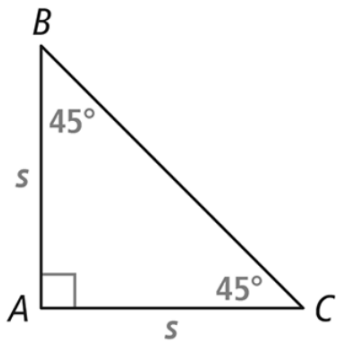
Enter your answer.



45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, the legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

If...



Then... $BC = s\sqrt{2}$

PROOF: SEE EXERCISE 18.

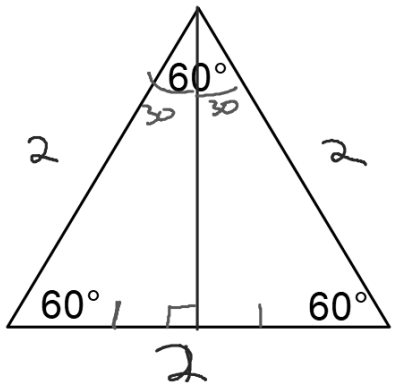
Special Right Triangles

30°- 60° - 90°

Short Leg – Half the Hypotenuse

Long Leg – Short Leg time $\sqrt{3}$

Hypotenuse -



$a^2 + b^2 = (2a)^2$

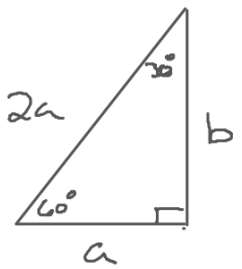
$a^2 + b^2 = 4a^2$

$-a^2 \qquad -a^2$

$b^2 = 3a^2$

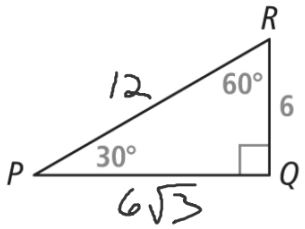
$b = \sqrt{3a^2}$

$= a\sqrt{3}$



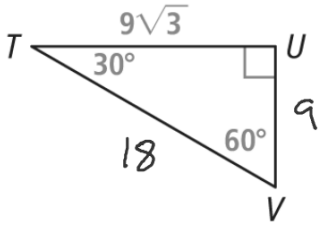
a. What are *PQ* and *PR*?

Enter your answer.



What are *UV* and *TV*?

ıter your answer.

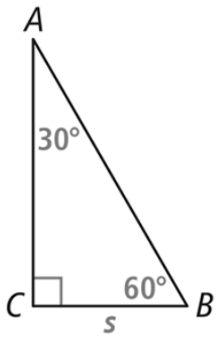


30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the short leg. The length of the long leg is $\sqrt{3}$ times the length of the short leg.

PROOF: SEE EXERCISE 19.

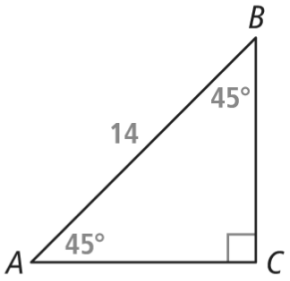
If...



Then... $AC = s\sqrt{3}$, $AB = 2s$

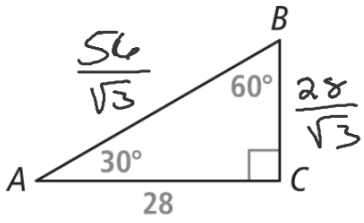
b. What are AC and BC?

$AC = BC = \frac{14}{\sqrt{2}}$



$\left(\frac{28}{\sqrt{3}}\right)\left(\frac{2}{1}\right)$

What are AB and BC?



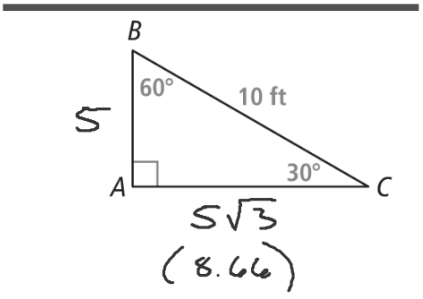
A. Alejandro needs to make both the horizontal and vertical supports, $\overline{AC}$ and $\overline{AB}$, for a ramp. Is one 12-foot board long enough for both supports? Explain.

└ NO more than 12'

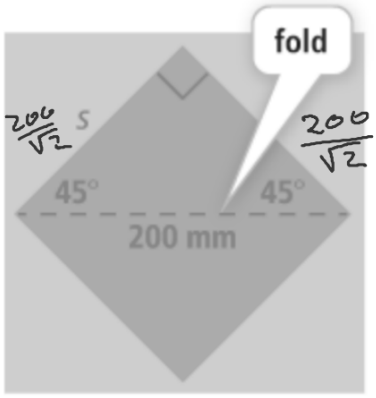
SOLUTION

$5 + 8.66$

$13.66 > 12$



B. Olivia starts an origami paper crane by making the 200-mm diagonal fold. What are the side length and area of the paper square?



$$A = \left(\frac{200}{\sqrt{2}}\right)^2$$

$$= \frac{40{,}000}{2}$$

$$= 20{,}000 \text{ mm}^2$$

If $RS = 35$, $ST = 37$, and $RT = 71$, is $\triangle RST$ a right triangle? Explain.

$2594 < 5041$

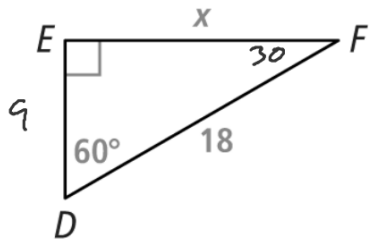
If $RS = 40$, $ST = 41$, and $RT = 11$, is $\triangle RST$ a right triangle? Explain.

$40^2 + 11^2 \stackrel{?}{=} 41^2$

If $RS = 20$, $ST = 21$, and $RT = 29$, is $\triangle RST$ a right triangle? Explain.

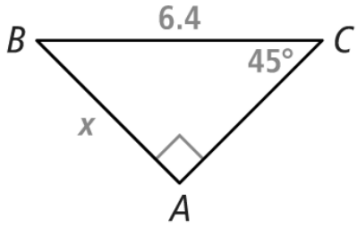
Find the value of x.

Enter your answer.



Find the value of x.

Enter your answer.



9. Charles wants to hang the pennant shown vertically between two windows that are 19 inches apart. Will the pennant fit? Explain.

Enter your answer.